SEMESTER EXAMINATION-2021
CLASS –B.Sc. V SEMESTER SUBJECT: MATHEMATICS
PAPER CODE: BMA-E503 PAPER TITLE: LINEAR ALGEBRA

Time: 3 hour **Max. Marks: 70**
Min. Pass: 40%

Note: Question Paper is divided into two sections: **A and B.** Attempt both the sections as per given instructions.

SECTION-A (SHORT ANSWER TYPE QUESTIONS)

Instructions: Answer any five questions in about 150 words each. Each question carries six marks. (5 X 6 = 30 Marks)

Question-1: Let W={$(x, y, z): x^2 = y$}. Is W is a subspace of $V_3(R)$.
Question-2: Is the set {(1,0), (2,3), (1,2)} linearly dependent?. Justify
Question-3: Show that the mapping $T: R^3 \to R^3$ defined by
$T(x, y, z) = (x + 2y,\ y + z, x)$ is linear.
Question-4: Prove that intersection of two subspaces W_1 and W_2 of a vector space V(F) is also a subspace of V(F).

Question-5: Define any two of the following with an example:
(i) Basis of a vector space
(ii) Linear functional
(iii) Dual Basis
Question-6: Find a linear transform $T: R^2 \to R^2$ such that T(2,3)=(4,5) and T(1,0)=(0,0).
Question-7: Show that a linear transformation $T: V \to W$ is one-one iff null set N(T) is the zero space $\{o_V\}$.
Question-8: Let $T: V_3(R) \to V_3(R)$ defined by:
$T(a, b, c) = (3a, a - b, 2a + b + c),\ \forall\ (a, b, c) \in V_3(R)$
Is T is invertible?. If so find T^{-1}
Question-9: Let A be an $n \times n$ matrix and λ an eigenvalue of A. Show that the set of all eigenvectors corresponding to λ, together with the zero vector, is a subspace of R^n.
Question-10: Prove that any two bases of a finite dimensional vector space have same number of elements

SECTION-B (LONG ANSWER TYPE QUESTIONS)

Instructions: Answer any FOUR questions in detail. Each question carries 10 marks. (4 X 10 = 40 Marks)

Question-11: Prove that a non-empty subset W of a vector space V(F) is a subspace of V if and only if :

$$a, b \in F \text{ and } \alpha, \beta \in W \Rightarrow a\alpha + b\beta \in W$$

Question-12: Show that the set $V = \{(x, y, z): x, y, z \in R\}$ is a vector space over the field R.

Question-13: If U and V be the vector spaces over the field F and T be a linear transform from U into V then Prove that:

Rank(T) + Nullity (T)= Dim U

Question-14: Prove that every n-dimensional vector space V(F) is isomorphic to V_n(F).

Question-15: Find the dual basis of the basis set B={(1,-1,3), (0,1,-1), (0,3,-2)} for $V_3(R)$.

Question-16: Let the linear map $T: V_3 \to V_2$ defined by $T(x, y, z) = (2x + y, 2y - x)$. Find the matrix of the transformation relative to bases $B_1 = \{(1,1,0), (1,0,1), (0,1,1)\}$ and $B_2 = \{(1,1), (1, -1)\}$

Question-17: If $W_1 \, and \, W_2$ are two sub spaces of vector space V(F) then prove that

$$dim(W_1 + W_2) = \dim W_1 + dim\, W_2 - \dim(W_1 \cap W_2)$$

Question-18: Find the eigen values and eigen vectors of the matrix: $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

Paper Code: BMA-E503