B. Sc. V SEMESTER

Semester Examination 2021
Subject: Mathematics
Paper Code: BMA-S504
Paper Name: COMBINATORIAL OPTIMIZATION

TIME: 3 Hrs **MAX. MARKS: 70**
Min. Pass % : 40

Note: This question paper is divided into two sections ***A*** and ***B***. Attempt all sections as per instructions.

Section-A (Short Answer Type Questions)

Note: Answer any **FIVE** questions in about 150 words each. Each question carries **SIX marks**.

1. Show that the intersection of two convex sets is again a convex set.
2. Write down the computational procedure of Simplex method.
3. Find the dual of the following primal problem

   Min. $Z = x_2+5x_3$, subject to, $x_1+x_2 \geq 5$, $2x_1+x_2+6x_3 \leq 6$, $x_1-x_2+3x_3=4$ and $x_1, x_2, x_3 \geq 0$.
4. Define symmetric and unsymmetric dual problem.
5. Define slack variables, surplus variables and artificial variables.
6. Discuss combinatorial optimization problem.
7. Show that the set of all convex combinations of finite number of points is a convex set.
8. Discuss local and global optimality.
9. What is degeneracy? Discuss a method to resolve degeneracy.
10. Discuss travelling salesman problem.

Section-B (Long Answer Type Questions)

NOTE: Answer any **FOUR** questions in detail. Each question carries **TEN marks**.

1. Prove that dual of the dual of a given primal is the primal itself.

2. Solve the following LPP by using Big-M method:

$$
\begin{aligned}
&Min\, Z = x_1 + x_2 \\
&s.t. \qquad 2x_1 + x_2 \geq 4 \\
&\qquad\quad x_1 + 7x_2 \geq 7 \qquad x_1, x_2 \geq 0
\end{aligned}
$$

3. Solve the following problem by dual simplex method.

   Min. Z = $2x_1+x_2$, subject to, $3x_1+x_2 \geq 3$, $4x_1+3x_2 \geq 6$, $x_1+2x_2 \geq 3$ and $x_1, x_2 \geq 0$.

4. Find the optimum integer solution of the following integer programming problem:

   Max. Z = x_1+x_2, subject to, $3x_1-2x_2 \leq 5$, $x_1 \leq 2$ and $x_1, x_2 \geq 0$ and are integers.

5. Discuss Cutting-Plane algorithms.

6. Use Branch and Bound technique to solve the following problem:

$$
\begin{aligned}
&Max. Z = 7x_1 + 9x_2 \\
&subject\, to\, -x_1 + 3x_2 \leq 6 \\
&\qquad 7x_1 + x_2 \leq 35 \qquad 0 \leq x_1, x_2 \leq 7 \\
&\qquad x_1, x_2 \text{ are integers.}
\end{aligned}
$$

7. Show that the set of all basic feasible solutions of a linear programming problem is a convex set.

8. Use two-phase method to solve the following problem:

$$
\begin{aligned}
&Max. Z = x_1 + x_2 \\
&subject\, to\;\; 2x_1 + x_2 \geq 4 \\
&\qquad x_1 + 7x_2 \geq 7 \qquad x_1, x_2 \geq 0.
\end{aligned}
$$

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