SEMESTER EXAMINATION – 2021

CLASS - M.Sc.

SUBJECT: MATHEMATICS

MMA-C114 : ABSTRACT ALGEBRA

Time: 3 hrs. **Max Marks: 70**

Min. Pass: 40%

Note: Question paper is divided into two sections: **A and B**. Attempt both the sections as per given instructions.

Section – A (Short Answer Type Questions)

Instructions: Answer any FIVE questions in about 150 words each. Each question carries six marks. (5X6=30 Marks)

1. Define a normal subgroup of a group. Let G be a group. Then prove that $H \subseteq G$ is normal iff every right coset of H is also a left coset of H.

2. Let G be a finite group such that $O(G)$ is 9. Then prove that G is an abelian group.

3. Let $f : G \longrightarrow G^*$ be a homomorphism from G onto G^*. Let K be the kernel of f. Then prove that $G^* \cong \frac{G}{K}$.

4. Prove that the subgroup of a solvable group is also solvable.

5. Let R be a commutative ring with multiplicative identity element. Let R does not have any proper ideal. Then show that R is a field.

6. Let S be an ideal of a commutative ring R with unity and R/S is a field then prove that S is maximal ideal.

7. Let L be a finite extension field of field K such that $[L : K] = m$. Let $a \in L$, then prove that $K(a)$ is a finite extension of K.

8. Let $f(x)$ be a polynomial of degree $n \geq 1$ in the ring of polynomials $F[x]$. Then show that n roots of polynomial $f(x)$ belong to a finite extension K of F. Also $[K : F] \leq n!$.

9. Let K be a normal extension of a field F of characteristic zero. If E is a subfield of K containing F such that E is a normal extension of F, then prove that
$$\sigma(E) \subseteq E \ \forall \sigma \in G(K, F).$$

10. Let K be an extension field of field F and $[K : F]$ is n. Then prove that
$$O(G(K,F)) \leq n.$$

. **Section - B (Long Answer Type Questions)**

Instructions: Answer any FOUR questions in detail. Each question carries ten marks.

(4 X 10 =40 Marks)

11. Let G be a finite group, p a prime, $p \mid O(G)$. Then show that G has a subgroup of order p.

12. Let $f : (Z,+) \longrightarrow (Z_4,+_4) \times (Z_6,+_6)$ defined by $f(n) = (n \bmod 4, n \bmod 6)$. Find range of f and kernel of f. Also check whether f is a homomorphism or not.

13. Let G be a group and H and K be distinct maximal subgroups of G. Then prove that $D = H \cap K$ is also normal subgroup of G. Further prove that $\frac{G}{K} \cong \frac{H}{D}$.

14. (a) Let H and K be normal subgroups of group G. Let $H \cap K = \{e\}$ and $G = HK$, then prove that G is internal direct product of H and K.
    (b) Find the unit element of ring Z / 12 Z.

15. Define vector space. Let $V(F)$ be a vector space generated by a set of n vectors. Let M be a linearly independent set of vectors in V. Prove that the number of vectors in M is less than equal to n.

16. Prove that for a polynomial $f(x) \in F[x]$, there exists a decomposition field, which is algebraic extension of F.

17. (a) Prove that F is an algebraic extension of K if F is an algebraic extension of E and E is an algebraic extension of K.
    (b) Find the irreducible polynomial of α over Q for $\alpha = \sqrt{2 + 4\sqrt{3}}$.

18. Let K be a normal extension of a field F of characteristic zero. Let E be any subfield of K such that E is normal extension of F, then prove that

$$[E:F] = \frac{O(G(K,F))}{O(G(K,E))}$$

and $G(K,E)$ is a normal subgroup of $G(K,F)$.

.